Journal of Applied Mechanics and Technical Physics, Vol. 48, No. 3, pp. 340–345, 2007

DYNAMICS AND RADIATION OF A SINGLE BUBBLE UNDER CONDITIONS OF ABNORMAL COMPRESSIBILITY OF A BUBBLY LIQUID

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UDC 532.5.013.2 + 532.593 + 532.529

An equation is proposed for the pulsation of a single cavity in an abnormally compressible bubbly liquid which is in pressure equilibrium and whose state is described by the Lyakhov equation. In the equilibrium case, this equation is significantly simplified. Numerical analysis is performed of the bubble dynamics and acoustic losses (the profile and amplitude of the radiation wave generated on the bubble wall from the side of the liquid). It is shown that as the volumetric gas concentration k_0 in the equilibrium bubbly medium increases, the degree of compression of the cavity by stationary shock wave decreases and its pulsations decrease considerably and disappear already at $k_0 = 3\%$. In the compression process, the cavity asymptotically reaches an equilibrium state that does not depend on the value of k_0 and is determined only by the shock-wave amplitude. The radiation wave takes the shape of a soliton whose amplitude is much smaller and whose width is considerably greater than the corresponding parameters in a single-phase liquid.

Key words: *dynamics, radiation, single bubble, abnormal compressibility.*

Introduction. Although theoretical studies of bubbly media have been performed for a long time, a "collective" velocity potential that would allow one to derive an equation for the pulsation of an individual bubble in a system of interacting bubbles has not been constructed. For example, in models such as the Iordanskii–Kogarko–van Wijngaarden (IKW) model, this interaction is taken into account indirectly, through the pressure field [1–3]. The main feature of the IKW model is that the bubbly medium is treated as a homogeneous medium in which averaged density, pressure, and velocity are determined. The state of the medium at each time is described by a system of relations, including the equations of state for the mixture and the liquid and gaseous components, which are closed by a kinetic equation — the Rayleigh equation for a single bubble, in which the pressure at infinity on the right side is replaced by the average pressure in the medium [4]. This means that, in essence, the IKW model and its numerical analogs do not consider bubbles and take into account the special property of the medium considered — the pulsation nature of the change in state and the peculiar transfer of the energy of the wave field to the kinetic energy and internal energy of the medium and back. If necessary, the model predicts what occurs with any bubble in the system at any point of the space studied. This concept of the interaction of the field and medium turned out to be adequate to real physical processes that occur not only in artificially produced bubbly systems during their interaction with shock waves [4–6] but also in liquid media with natural microinhomogeneities, in which dynamic loading by rarefaction waves (phases) leads to the occurrence of cavitation processes [7].

Garipov [8] was apparently one of the first to undertake an attempt to construct the "collective" potential. A simple model for the interaction of two bubbles and the pressure waves radiated by them was studied by Fujikawa and Takahira [9], who concluded that the compressibility of the liquid phase plays an important role. This conclusion is fairly obvious since in [9] the radiation of bubbles was considered. It has been shown that for some special combinations of the initial radii of two bubbles and the initial gas pressures in them, the smaller bubble generates high pressure pulses with amplitudes six times higher than the radiation amplitude of a single bubble. This effect

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disappears if the bubbles have the same size. It can be assumed that the mechanism of this phenomenon is due to the overcompression of the smaller bubble by the shock wave generated by the larger bubble. Gasenko et al. [10] proposed a model for perturbation propagation that takes into account the compressibility of the carrier phase, i.e., corresponds to both (high- and low-frequency) branches of the dispersion curve. We note that although the two-phase model takes into account the "collective" bubble pulsations indirectly, through the average pressure field, the states of both phases "exist" separately. Thus, bubble pulsations under the action of the average pressure and its losses due to radiation are considered in the carrier single-phase medium. In this situation, it is necessary to elucidate whether all advantages of the averaging model are taken into account. As is known, a bubbly medium is a homogeneous system that has an important physical feature — abnormal compressibility, which is manifested in abnormally low velocities of perturbation propagation. The question arises: should this feature be manifested in the behavior of the "collective" bubble in this system? In the present paper, a model is constructed that describes the dynamics and radiation of a single bubble under conditions of abnormal compressibility of a bubbly liquid. The bubble behavior in this medium is estimated numerically.

Formulation of the Problem. The dynamics of a single bubble in a compressible bubbly liquid is considered. The flow is potential, $u = -\nabla\varphi$, and the velocity potential in a compressible bubbly liquid has the standard form

$$\varphi = \Phi(t - r/c_{\rm b})/r, \tag{1}$$

where $c_{\rm b}^2 = c_l^2/(1 + k_0\bar{B})$ is the squared sound velocity in the unperturbed bubbly medium k_0 is the volume concentration of the gas phase, and $\bar{B} = nB/p_0$ (n and B are constants in the equation of state of the liquid component). Then, the mass velocity is defined by the expression

$$u = \Phi/r^2 + \Phi'/(c_{\rm b}r), \tag{2}$$

where the prime denotes differentiation with respect to $\zeta = t - r/c_{\rm b}$, and the Cauchy–Lagrange integral, with allowance for (1), becomes

$$\Phi' = r(\omega + u^2/2) \tag{3}$$

($\omega = \int dp/\rho$ is the enthalpy). Introducing the function $\Omega = \omega + u^2/2$, from Eqs. (2) and (3), we find the expressions for Φ and its derivative:

$$\Phi = r^2(u - \Omega/c_{\rm b}), \qquad \Phi' = r^2(u_t - \Omega_t/c_{\rm b}) = r^2[u_t - (\omega_t + uu_t)/c_{\rm b}]. \tag{4}$$

Here $\Phi_t = \Phi'$. From the conservation laws

$$u_r + \frac{2u}{r} = -\frac{1}{c_{\rm b}^2}\frac{d\omega}{dt}, \qquad \frac{\partial\omega}{\partial r} = -\frac{du}{dt},$$

we find the derivatives u_t and ω_t:

$$\frac{du}{dt} + \frac{2u^2}{r} + \frac{u}{c_{\rm b}^2}\frac{d\omega}{dt} = u_t, \qquad \omega_t = u\frac{du}{dt} + \frac{d\omega}{dt}.$$

Substituting these derivatives into (4) and using (3), we obtain

$$r\Big(1 - \frac{2u}{c_{\rm b}}\Big)\frac{du}{dt} + \frac{3}{2}u^2\Big(1 - \frac{4u}{3c_{\rm b}}\Big) = \omega + \frac{r}{c_{\rm b}}\Big(1 - \frac{u}{c_{\rm b}} + \frac{u^2}{c_{\rm b}^2}\Big)\frac{d\omega}{dt}. \tag{5}$$

If $r = R$, $u = \dot{R}$, and $\omega = H$ on the cavity wall, Eq. (5) becomes

$$R\Big(1 - \frac{2\dot{R}}{c_{\rm b}}\Big)\ddot{R} + \frac{3\dot{R}^2}{2}\Big(1 - \frac{4\dot{R}}{3c_{\rm b}}\Big) = H + \frac{R}{c_{\rm b}}\Big(1 - \frac{\dot{R}}{c_{\rm b}} + \frac{\dot{R}^2}{c_{\rm b}^2}\Big)\frac{dH}{dt}. \tag{6}$$

To determine the enthalpy H on the cavity wall, we assume that outside the cavity, the bubbly medium is in pressure equilibrium. Then, for the isothermal case, provided that the phases are in pressure equilibrium ($pk = p_0k_0$), the equation of state in the form of the Lyakhov equation [11]

$$\rho_0/\rho = k(p/p_0)^{-1/\gamma} + (1 - k)P^{-1/n},$$

where $P = 1 + n(p - p_0)/(\rho_l c_l^2)$ is on the order of unity, becomes

$$\rho_0/\rho = k_0(p_0/p)^2 + 1 - k_0(p_0/p).$$

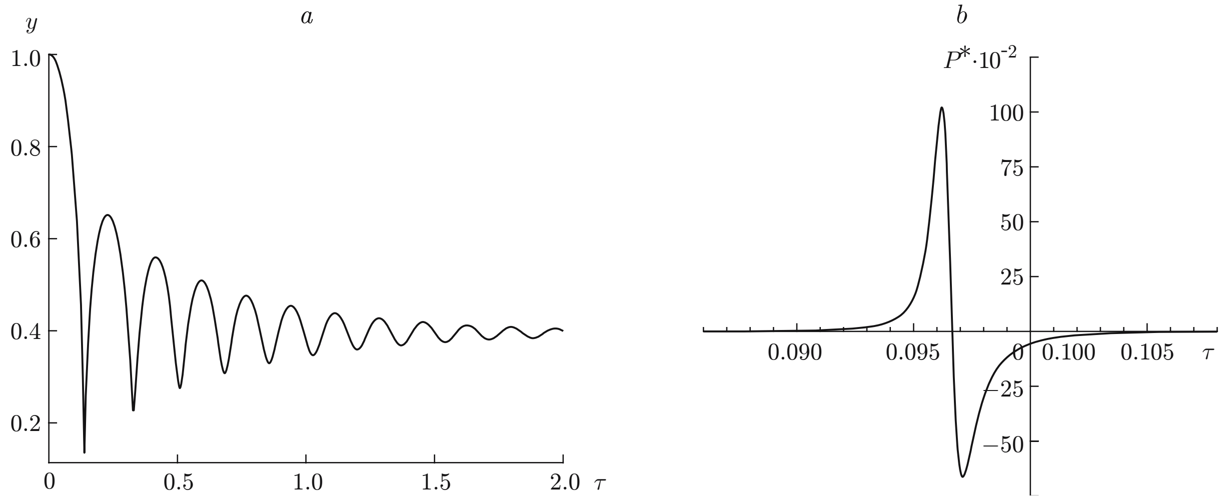

Fig. 1. Dynamics (a) and radiation profile (b) of a single bubble in a single-phase liquid at $k_0 = 0$.

The enthalpy on the cavity wall is defined by the integral

$$H = \frac{1}{\rho_0} \int\limits_{p_\infty}^{p(R)} \frac{\rho_0}{\rho}\, dp = \frac{1}{\rho_0} \int\limits_{p_\infty}^{p(R)} \Big[k_0 \Big(\frac{p_0}{p}\Big)^2 + 1 - k_0 \frac{p_0}{p}\Big]\, dp,$$

and, hence,

$$H = \frac{p(R) - p_\infty}{\rho_l(1 - k_0)} + \frac{p_0 k_0}{\rho_l(1 - k_0)} \Big[\frac{p_0}{p_\infty} - \frac{p_0}{p(R)} - \ln \frac{p(R)}{p_\infty}\Big].$$

In this expression, the second term can be ignored. As a result, we have

$$H = \frac{p(R) - p_\infty}{\rho_l(1 - k_0)}.$$

In view of the derivative of the enthalpy, the equation describing the cavity pulsation (6) in the bubbly medium finally becomes

$$R\Big(1 - \frac{2\dot{R}}{c_\text{b}}\Big)\ddot{R} + \frac{3}{2}\dot{R}^2\Big(1 - \frac{4\dot{R}}{3c_\text{b}}\Big) = \frac{p(R)}{\rho_l(1 - k_0)}\Big[1 - 3\gamma\Big(\frac{\dot{R}}{c_\text{b}} - \frac{\dot{R}^2}{c_\text{b}^2} + \frac{\dot{R}^3}{c_\text{b}^3}\Big)\Big] - \frac{p_\infty}{\rho_l(1 - k_0)}. \tag{7}$$

Obviously, for small values of the volumetric concentration k_0 the bubbly media containing the spherical cavity studied, it can be assumed that the external pressure (shock-wave amplitude) p_∞ does not depend on k_0 and that the pressure equilibrium in this medium is established instantaneously. Acoustic losses (radiation) are taken into account by the term

$$P^* = -\frac{3\gamma p(R)}{p_0(1 - k_0)}\Big(\frac{\dot{R}}{c_\text{b}} - \frac{\dot{R}^2}{c_\text{b}^2} + \frac{\dot{R}^3}{c_\text{b}^3}\Big)$$

on the right side of Eq. (7). The role of the acoustic corrections $\dot{R}/c_\text{b}$ on the left side of the equation is easily determined as follows. Multiplication of both sides of the equality into $2R^2$ reduces the left side of Eq. (7) to the form

$$\frac{d}{dR}\, R^3 \dot{R}^2 \Big(1 - \frac{4}{3}\,\frac{\dot{R}}{c_\text{b}}\Big),$$

which allows the first integral of Eq. (7) to be written. Solution of Eq. (7) ignoring radiation (ignoring the term with the derivative dH/dt) shows that the correction given above influences only the period of bubble pulsation. We note that by analogy with the formulation of the problem of the interaction two identical bubbles considered in [9], as a first approximation, one can find the potential in the vicinity of the central bubble of N bubbles that

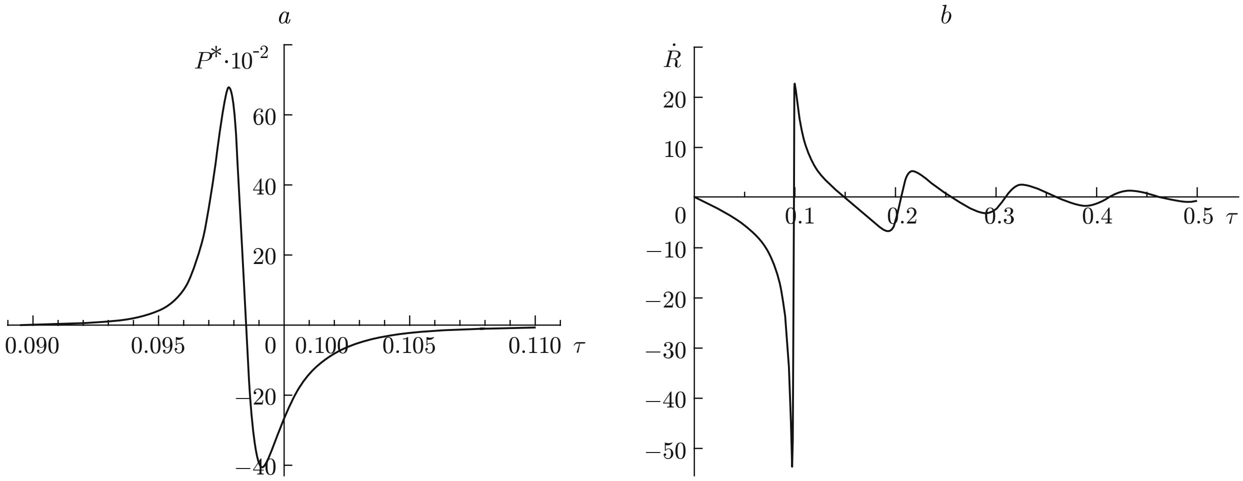

Fig. 2. Radiation profile (a) and radial expansion velocity (b) of a single bubble in an equilibrium bubbly liquid at $k_0 = 0.01\%$.

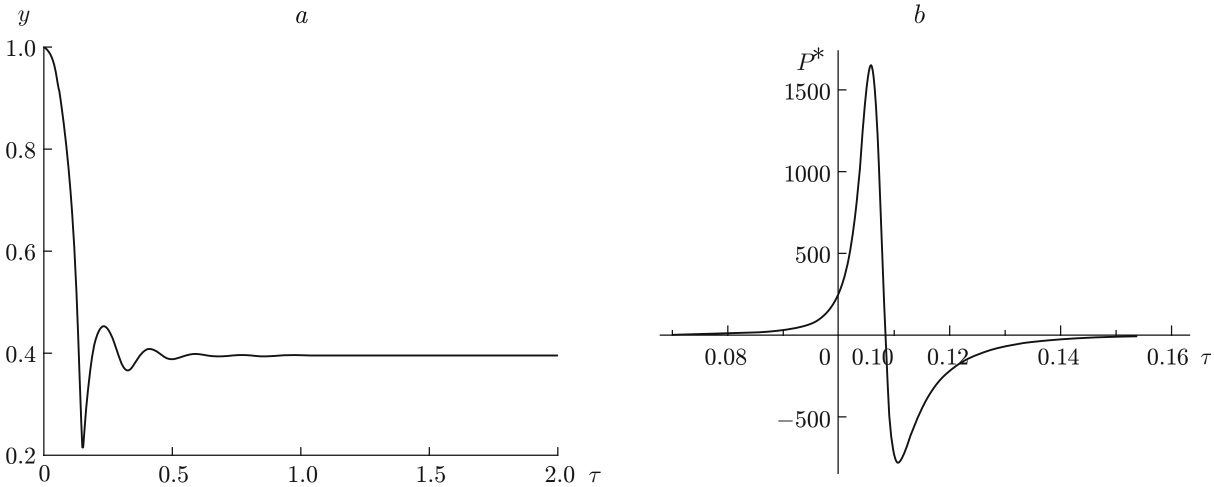

Fig. 3. Dynamics (a) and radiation profile (b) of a single bubble at $k_0 = 0.1\%$.

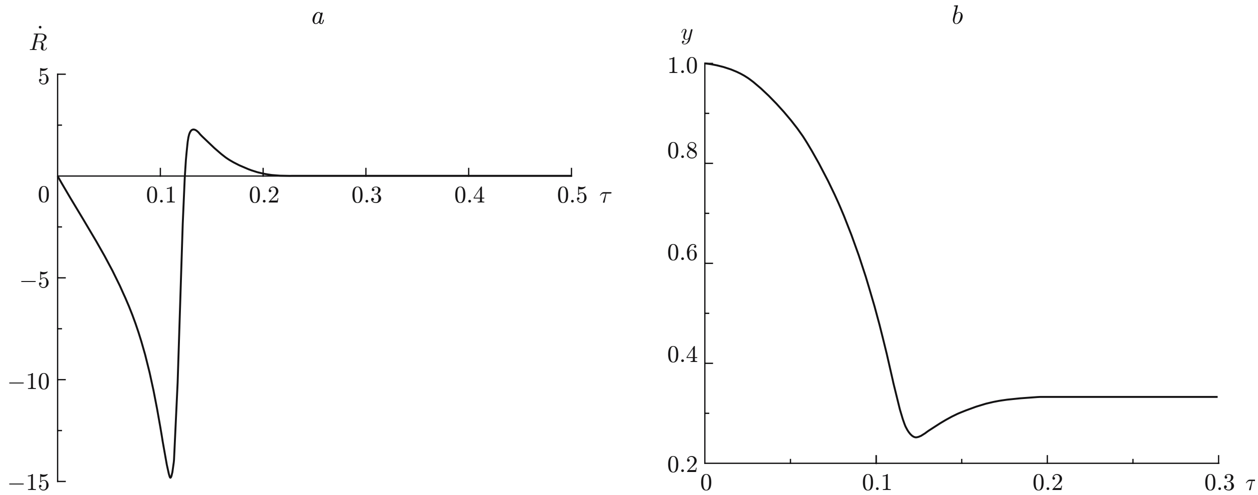

Fig. 4. Radial velocity (a) and dynamics (b) of a single bubble for $k_0 = 0.5\%$.

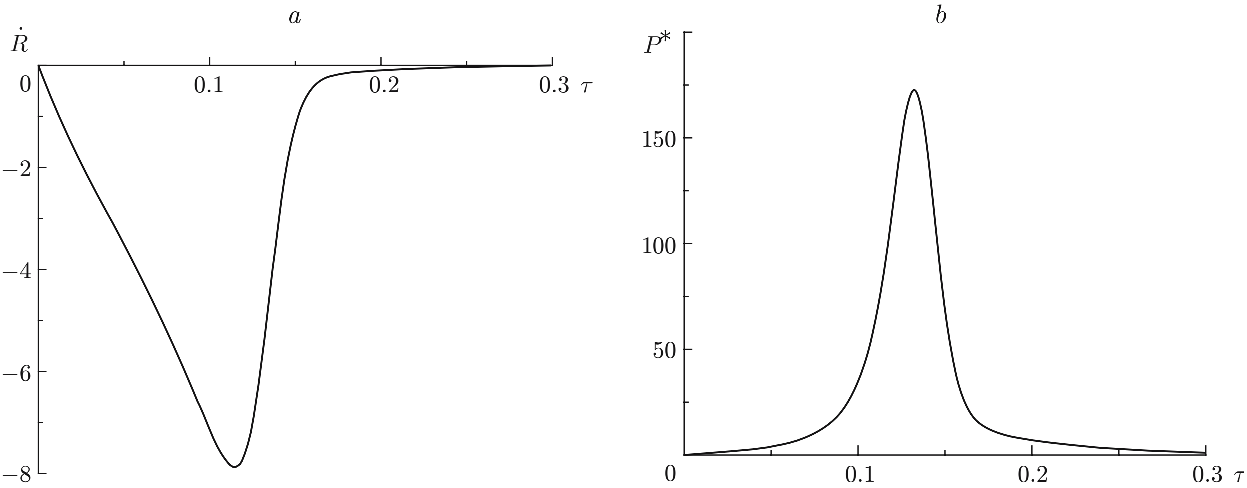

Fig. 5. Radial velocity (a) and radiation profile (b) of a single bubble at $k_0 = 3\%$.

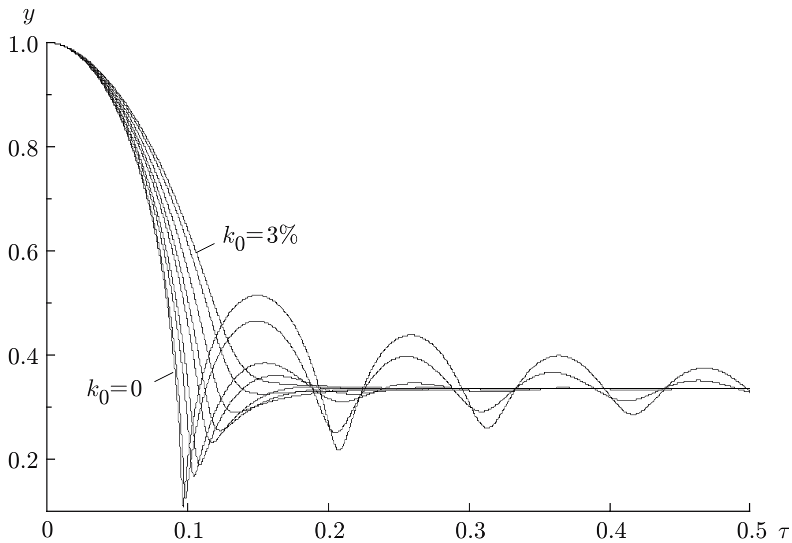

Fig. 6. Dynamics of a single bubble in an equilibrium bubbly liquid at $k_0 = 0$, 0.01, 0.05, 0.1, 0.3, 0.5, 1.0, 2.0, and 3.0%.

are uniformly distributed at the sites of a lattice of cubic elements. In the equation describing bubble pulsations in such a lattice in an incompressible liquid, the first integral has the form

$$\frac{d}{dR}\, R^3 \dot{R}^2 \Big(1 + \alpha N\, \frac{R}{l}\Big) = 2R^2\, \frac{p(R) - p_\infty}{\rho_l},$$

where $\alpha < 1$; l is the lattice-element parameter related to the volumetric concentration by the formula $k_0 = (R_0/l)^3 4\pi/3$. It can be concluded that interaction of the bubbles results in a reduction in their pulsation frequency with increasing k_0.

Results of Numerical Analysis. Below, we give the results of calculating the dynamics of the relative radius of a bubble $y = R/R_0$, its radial velocity $y_\tau = dy/d\tau$, and the radiation P^* (acoustic losses) as a function of the dimensionless time $\tau = t\sqrt{p_0/\rho_l}/R_0$ for an amplitude of the external stationary pressure $p_\infty = 10$ MPa, a hydrostatic pressure $p_0 = 0.1$ MPa, and initial bubble radius R_0. Figure 1 shows the calculated dynamics and radiation profile of bubble in a single-phase liquid. We note that the radiation wave is determined primarily by the first pulsation of the radial velocity (by the acceleration dynamics of the cavity wall). The supply of an insignificant amount of a gas phase ($k_0 = 0.01\%$) leads to a significant change in the radiation parameters (Fig. 2). In this case, the pressure amplitude is almost halved.

A further increase in the volume of the gas phase (to 0.1%) leads to a decrease in the degree of compression and an increase in the time of compression to the minimum radius (Fig. 3). This result is obvious: the pulsations of the "interacting" bubbles become lower-frequency. An increase in the volume fraction of the gas leads to a reduction in the number of pulsations, resulting in faster attainment of the equilibrium state of the bubble (Fig. 3). Figure 4 shows the radial velocity and dynamics of a single bubble in an equilibrium bubbly liquid at $k_0 = 0.5\%$. From the dependences presented in Fig. 4, it follows that as the gas-phase concentration in the surrounding liquid increases to a value $k_0 = 0.5\%$, the bubble performs only one pulsation. At $k_0 = 3\%$, the damping fluctuations disappear and the bubble asymptotically reaches an equilibrium state already during the first compression (Fig. 5). It is easy to see that for this value of k_0, the radiation profile takes the shape of a soliton whose amplitude is approximately 50 times smaller than the corresponding value for the single-phase liquid (see Fig. 5). In this case, the bubble dynamics without overcompression (inertial terms do not "work") is described by a function that tends asymptotically to the equilibrium state. We note that this state does not depend on the value of k_0 and is determined only by the external pressure. Figure 6 shows a curve of bubble collapse under the action of a shock wave with a constant profile and an amplitude of 10 MPa at $k_0 = 0$–$3 \cdot 10^{-2}$. According to the results of the numerical analysis, starting with values of $k_0 \approx 3\%$, the process of adiabatic compression of the cavity reaches a peculiar threshold where dynamic equilibrium is established between the radiation and the inertia of the attached mass (see Fig. 5), whose decrease in the energetic balance is compensated by increased acoustic losses. There occurs a peculiar regime of inertia-free compression (unlimited cumulation) to the cavity radius corresponding to the equilibrium state.

Thus, the calculations performed show that the proposed model allows one to analyze the bubble dynamics in an abnormally compressible liquid and to study the radiation structure and parameters.

This work was supported by the Russian Foundation for Basic Research (a Grant No. 06-01-00317) and the Integration project of the Siberian Division of the Russian Academy of Sciences No. 4.13.4.

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